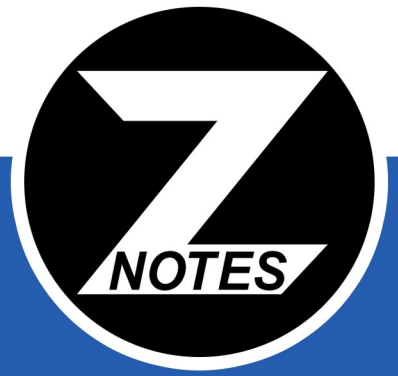




Updated to 2017-18 Syllabus

CIE IGCSE MATHS 0580

SUMMARIZED NOTES ON THE EXTENDED SYLLABUS

TABLE OF CONTENTS

2 | CHAPTER 1
Number

3 | CHAPTER 2
Algebra & Graphs

4 | CHAPTER 3
Geometry

6 | CHAPTER 4
Mensuration

7 | CHAPTER 5
Coordinate Geometry

7 | CHAPTER 6
Trigonometry

8 | CHAPTER 7
Matrices & Transformations

9 | CHAPTER 8
Probability

9 | CHAPTER 9
Statistics

1. NUMBER

• Natural numbers:

- used for counting purposes
- made up of all possible rational & irrational numbers

• Integer: a whole number

• Prime numbers:

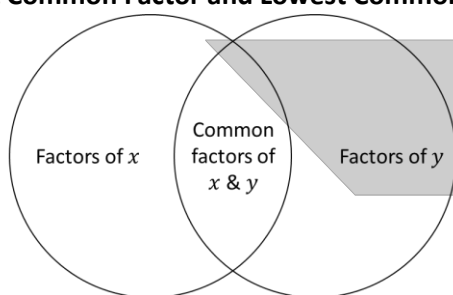
- divisible only by itself and one
- 1 is not a prime number

• Rational numbers: can be written as a fraction

• Irrational numbers: cannot be written as a fraction e.g. π

1.1 HCF and LCM

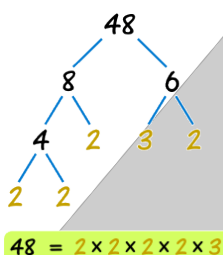
• Highest Common Factor and Lowest Common Multiple:



- HCF = product of common factors of x and y
- LCM = product of all items in Venn diagram

• Prime Factorization: finding which prime numbers

- multiply together to make the original number



1.2 Sets

• Definition of sets e.g.

- $A = \{x: x \text{ is a natural number}\}$
- $B = \{(x, y): y = mx + c\}$
- $C = \{x: a \leq x \leq b\}$
- $D = \{a, b, c, \dots\}$

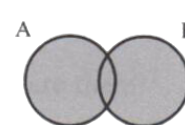
Notation:

- $n(A)$ = no. of elements in A
- $\in$ = ...is an element of...
- $\notin$ = ...is not an element of...
- A' = compliment of set A
- $\emptyset$ or $\{\}$ = empty set
- $\mathbb{U}$ = Universal set
- $A \cup B$ = union of A and B
- $A \cap B$ = intersection of A and B
- $A \subseteq B$ = A is a subset of B
- $A \subset B$ = A is a proper subset of B
- $A \not\subseteq B$ = A is not a subset of B
- $A \not\subset B$ = A is not a proper subset of B

Set representations:



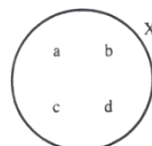
$A \cap B$ is shaded



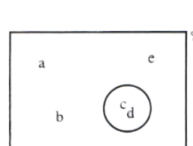
$A \cup B$ is shaded



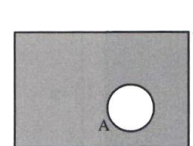
A is a subset of B



$b \in X$



$\mathbb{U} = \{a, b, c, d, e\}$



A' is shaded

1.3 Indices

- $n(A)$ = no. of elements in A
- $\in$ = ...is an element of...
- $\notin$ = ...is not an element of...
- A' = compliment of set A
- $\emptyset$ or $\{\}$ = empty set
- $\mathbb{U}$ = Universal set
- $A \cup B$ = union of A and B
- $A \cap B$ = intersection of A and B
- $A \subseteq B$ = A is a subset of B
- $A \subset B$ = A is a proper subset of B
- $A \not\subseteq B$ = A is not a subset of B
- $A \not\subset B$ = A is not a proper subset of B

Standard form:

- $10^4 = 10000$
- $10^3 = 1000$
- $10^2 = 100$
- $10^1 = 10$
- $10^0 = 1$
- $10^{-1} = 0.1$
- $10^{-2} = 0.01$
- $10^{-3} = 0.001$
- $10^{-4} = 0.0001$
- $10^{-5} = 0.00001$

Limits of accuracy:

- The degree of rounding of a number
- E.g. 2.1 to 1 d.p. $2.05 \leq x < 2.15$

1.4 Ratio & Proportion

• Ratio: used to describe a fraction

- e.g. 3 : 1

• Foreign exchange: money changed from one currency to another using proportion

- E.g. Convert \$22.50 to Dinars
- $\$1 : 0.30\text{KD}$
- $\$22.50 : 6.75\text{KD}$

• Map scales: using proportion to work out map scales

- 1km = 1000m
- 1m = 100cm
- 1cm = 10mm

- **Direct variation:** y is proportional to x

$$y \propto x \quad y = kx$$

- **Inverse variation:** y is inversely proportional to x

$$y \propto \frac{1}{x} \quad y = \frac{k}{x}$$

1.5 Percentages

- **Percentage:**

- Convenient way of expressing fractions
- Percent means per 100

- **Percentage increase or decrease:**

$$\text{Percentage increase} = \frac{\text{Actual Increase}}{\text{Original Amount}}$$

- **Simple interest:**

$$I = \frac{PRT}{100}$$

P = Principal R = Rate of Interest T = Period of Time

- **Compound interest:**

$$A = P \left(1 + \frac{R}{100} \right)^n$$

P = Principal R = Rate of Interest n = Period of Time

1.6 Speed, Distance & Time

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

- **Units of speed:** km/hr m/s

- **Units of distance:** km m

- **Units of time:** hr sec

$$\text{km/hr} \times \frac{5}{18} = \text{m/sec}$$

$$\text{m/sec} \times \frac{18}{5} = \text{km/hr}$$

2. ALGEBRA & GRAPHS

2.1 Factorisation

- **Common factors:**

$$3x^2 + 6x$$

$$3x(x + 2)$$

- **Difference of two squares:**

$$25 - x^2$$

$$(5 + x)(5 - x)$$

- **Group factorization:**

$$4d + ac + ad + 4c$$

$$4(d + c) + a(c + d)$$

$$(4 + a)(c + d)$$

- **Trinomial:**

$$x^2 + 14x + 24$$

$$x^2 + 12x + 2x + 24$$

$$x(x + 12) + 2(x + 12)$$

$$(x + 2)(x + 12)$$

2.2 Quadratic Factorization

- **General equation:**

$$ax^2 + bx + c = 0$$

- **Solve quadratics by:**

- Trinomial factorization
- Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- When question says "give your answer to two decimal places", **use formula!**

2.3 Simultaneous Equations

- Simultaneous linear equations can be solved either by substitution or elimination
- Simultaneous linear and non-linear equations are generally solved by substitution as follows:
 - Step 1: obtain an equation in one unknown and solve this equation
 - Step 2: substitute the results from step 1 into the linear equation to find the other unknown
- The points of intersection of two graphs are given by the solution of their simultaneous equations

2.4 Inequalities

- Solve like equations
- Multiplying or dividing by negative $\Rightarrow$ switch sign

$$\frac{y}{-3} \geq -7$$

$$y \leq -7 \times -3$$

$$y \leq 21$$

- When two inequalities present, split into two

$$x < 3x - 1 < 2x + 7$$

$$x < 3x - 1$$

$$3x - 1 < 2x + 7$$

$$x > -\frac{1}{2}$$

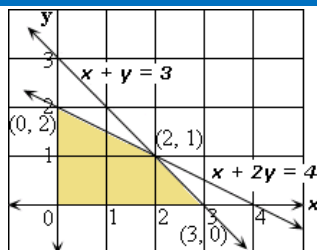
$$x < 8$$

2.4 Linear Programming

- For strict inequalities ($<$, $>$) use broken line
- For non-strict inequalities ($\leq$, $\geq$) use solid line

• Steps to solve:

- Interpret $y = mx + c$
- Draw straight line graphs
- Shade
- Solve



2.5 Sequences

- **Linear sequences:** Find common difference e.g. 3 then multiply by n and work out what needs to be added

- **Quadratic sequences:**

- Format: $an^2 + bn + c$



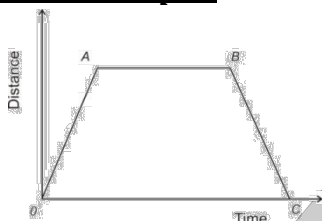
- Work out the values and then place into formula to work out n th term formula

- **Geometric progression:** sequence where term has been multiplied by a constant to form next term

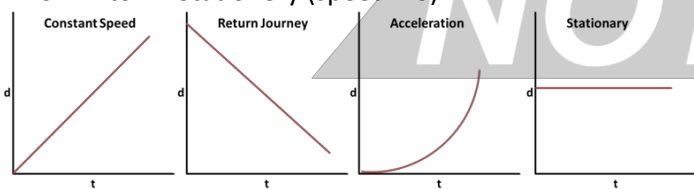
$$n\text{th term of G.P.} = ar^{(n-1)}$$

- $a = 1^{\text{st}}$ term $r = \text{common difference}$

2.6 Distance-Time Graphs

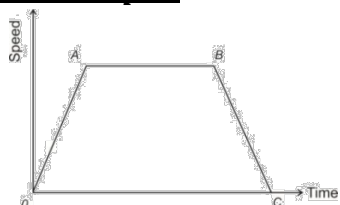


- From O to A : Uniform speed
- From B to C : Uniform speed (return journey)
- From A to B : Stationary (speed = 0)

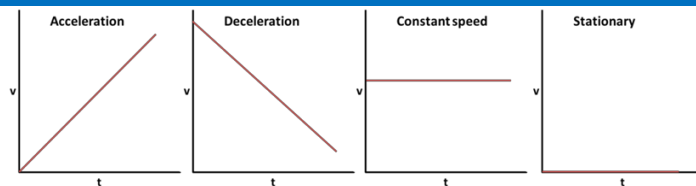


- Gradient = speed

2.7 Speed-Time Graphs



- From O to A : Uniform speed
- From A to B : Constant speed (acceleration = 0)
- From B to C : Uniform deceleration / retardation



- Area under a graph = distance travelled.
- Gradient = acceleration.
- If the acceleration is negative, it is called deceleration or retardation. (moving body is slowing down.)

2.8 Functions

- **Function notation:**

- $f: x \rightarrow 2x - 1$
- Function f such that x maps onto $2x - 1$

- **Composite function:** Given two functions $f(x)$ and $g(x)$, the composite function of f and g is the function which maps x onto $f(g(x))$

- $f(2)$

- Substitute $x = 2$ and solve for $f(x)$

- $fg(x)$

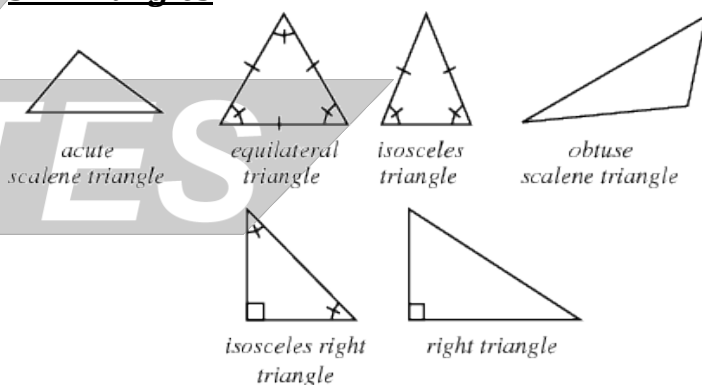
- Substitute $x = g(x)$

- $f^{-1}(x)$

- Let $y = f(x)$ and make x the subject

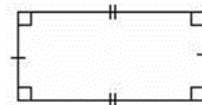
3. GEOMETRY

3.1 Triangles

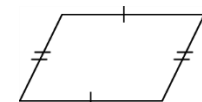


3.2 Quadrilaterals

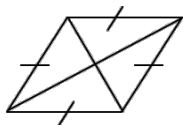
- **Rectangle:** Opposite sides parallel and equal, all angles 90° , diagonals bisect each other.



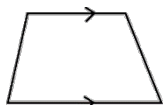
- **Parallelogram:** Opposite sides parallel and equal, opposite angles equal, diagonals bisect each other



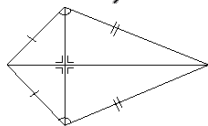
- **Rhombus:** A parallelogram with all sides equal, opposite angles equal, diagonals bisect each other



- **Trapezium:** One pair of sides parallel

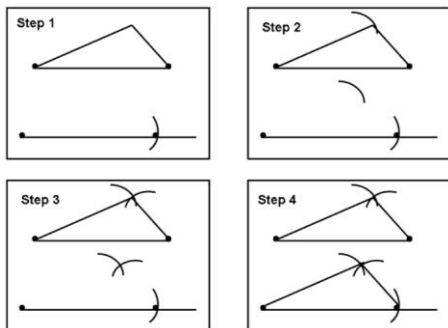


- **Kite:** Two pairs of adjacent sides equal, diagonals meet at right angles bisecting one of them

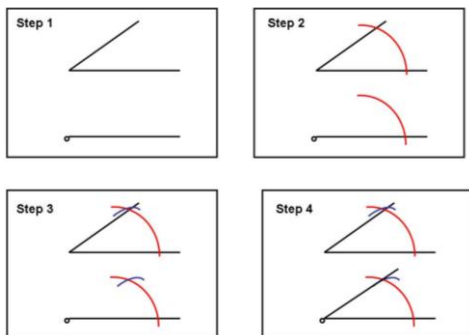


3.3 Construction

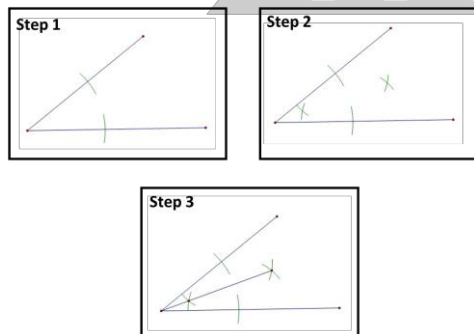
- **Constructing triangles:**



- **Perpendicular bisector:**



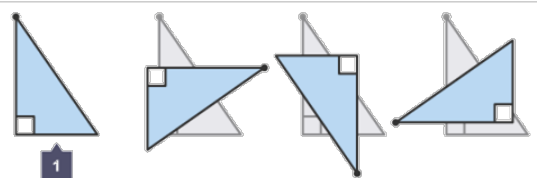
- **Angle bisector:**



3.4 Symmetry

- A **line of symmetry** divides a two-dimensional shape into two congruent (identical) shapes.
- A **plane of symmetry** divides a three-dimensional shape into two congruent solid shapes.

- The number of times shape fits its outline during a complete revolution is called the order of **rotational symmetry**.



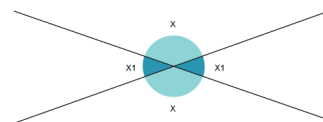
Shape	Number of Lines of Symmetry	Rotational Symmetry Order
Square	4	4
Rectangle	2	2
Parallelogram	0	2
Rhombus	2	2
Trapezium	0	1
Kite	1	1
Equilateral triangle	3	3
Regular hexagon	6	6

- **Properties of circles:**

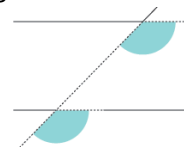
- Equal chords are equidistant from the centre
- The perpendicular bisector of a chord passes through the centre
- Tangents from an external point are equal in length

3.5 Polygons

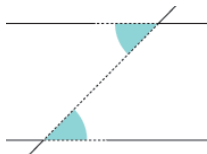
- Sum of angles at a point = 360
- Angles on a straight line = 180
- Sum of angles in a triangle = 180
- For regular polygon
 - External angles = $\frac{360}{n}$
 - Internal angles = $180 - \frac{360}{n}$
- For irregular polygon:
 - Sum of exterior angles = 360
 - Sum of interior angles = $180(n-2)$
- Vertically opposite angles



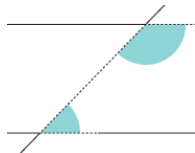
- Corresponding angles



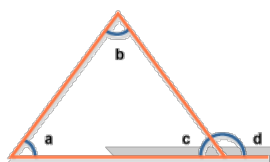
- Alternate angles



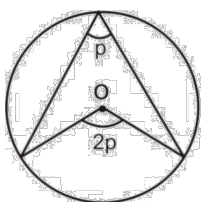
- Co-interior angles



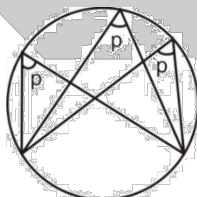
- Exterior angle = sum of interior opposite $\angle$



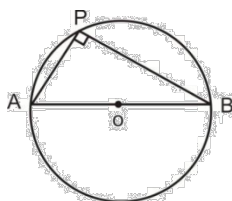
3.6 Circle Theorem



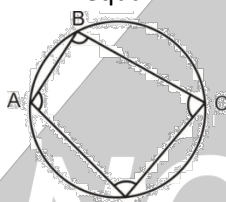
Angle at centre = twice angle on circumference



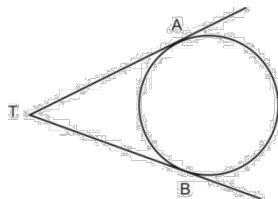
Angle subtended by same arc at circumference are equal



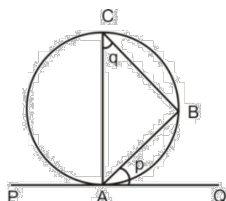
Angles in semicircle are 90°



Opposite angles in a cyclic quadrilateral = 180°



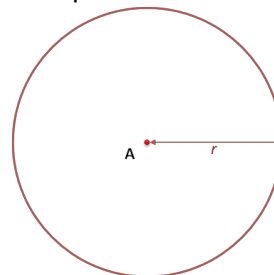
Tangents from one point are equal $\angle$ between tangent and radius is 90°



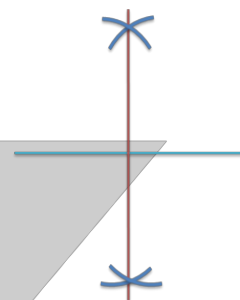
Alternate segment theorem

3.7 Loci

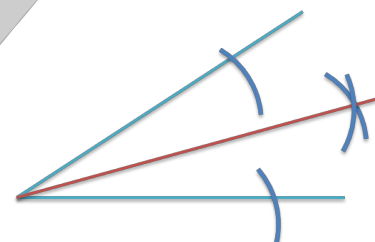
- The locus of points equidistant from a point is a circle



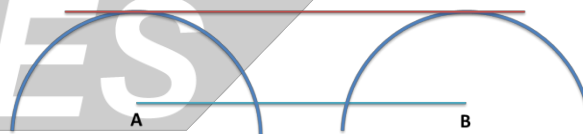
- The locus of points equidistant between two points is a perpendicular bisector



- The locus of points equidistant between two lines is an angle bisector



- The locus of points equidistant (along) from a line is a parallel line



4. MENSURATION

4.1 Area

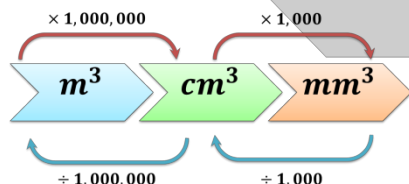
- Parallelogram = $b \times h$ OR $ab \sin \theta$
- Triangle = $\frac{1}{2} b \times h$
- Trapezium = $\frac{1}{2} (a + b)h$
- Circle = πr^2
- Sector = $\pi r^2 \times \frac{\theta}{360}$

4.2 Volume and Surface Area

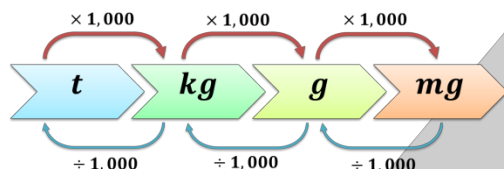
- Cylinder
 - Curved surface area = $2\pi rh$
 - Volume = $\pi r^2 h$
- Cone
 - Curved surface area = πrl
 - Volume = $\frac{1}{3}(\pi r^2 h)$
- Sphere
 - Surface area = $4\pi r^2$
 - Volume = $\frac{4}{3}\pi r^3$
- Hemisphere
 - Surface area = $2\pi r^2$
 - Volume = $\frac{2}{3}\pi r^3$

4.3 Units

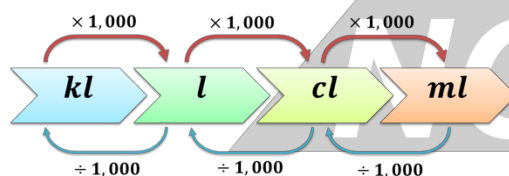
• Volume:



• Mass:



• Capacity:



• Connecting volume and capacity:

- $1ml = 1cm^3$
- $1kl = 1m^3$

• Density = $\frac{Mass}{Volume}$

5. COORDINATE GEOMETRY

5.1 Graphs

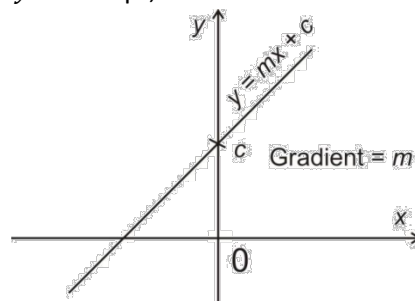
• Gradient of a Straight Line:

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

• Equation of Line:

$$y = mx + c$$

- Find the gradient, m
- Find the y-intercept, c



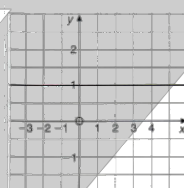
• Midpoint of Graph:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

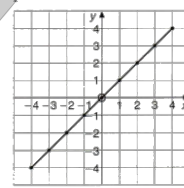
• Length between two points:

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

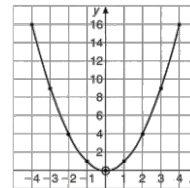
5.2 Sketching Graphs



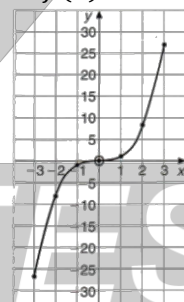
$$f(x) = 1$$



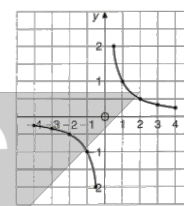
$$f(x) = x$$



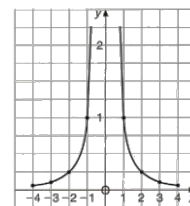
$$f(x) = x^2$$



$$f(x) = x^3$$



$$f(x) = \frac{1}{x}$$



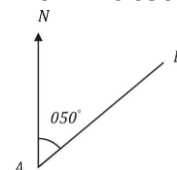
$$f(x) = \frac{1}{x^2}$$

6. TRIGONOMETRY

6.1 Bearings

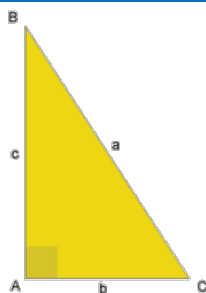
• The bearing of a point B from another point A is:

- An angle measured from the north at A.
- In a clockwise direction.
- Written as three-figure number (i.e. from 000° to 360°)
- e.g. The bearing of B from A is 050°



6.2 Pythagoras Theorem

- To find hypotenuse
 - $a^2 + b^2 = c^2$
- To find one of the shorter sides
 - $a^2 = c^2 - b^2$
 - $b^2 = c^2 - a^2$

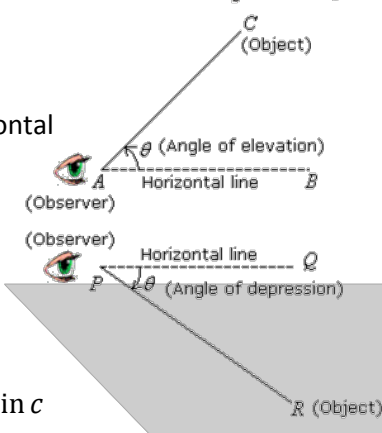


• Angle of elevation:

- Angle above the horizontal line.

• Angle of depression:

- Angle below the horizontal line.



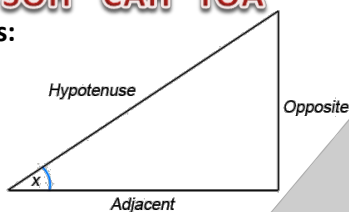
- Area of a triangle: $\frac{1}{2}ab \sin c$

6.3 Ratios

SOH CAH TOA

• Right angled triangles:

- $\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$
- $\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$
- $\tan x = \frac{\text{opposite}}{\text{adjacent}}$



6.4 Sine & Cosine Rules

• Sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- One pair of information needed

• Cosine rule

- To find the angle given 3 sides

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

- To find side given angle and two sides

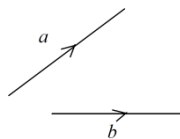
$$a^2 = b^2 + c^2 - 2bc \cos A$$

7. MATRICES & TRANSFORMATION

7.1 Vector

- A vector quantity has both magnitude and direction.

- E.g. Vectors a and b represented by the line segments can be added using the parallelogram rule or the nose-to-tail method.



• Multiplication by a scalar:

- A scalar quantity has a magnitude but no direction
- The negative sign reverses the direction of the vector

• Column vector:

- Top number is the horizontal component and bottom number is the vertical component

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

• Parallel vectors:

- Vectors are parallel if they have the same direction
- In general the vector $k\begin{pmatrix} a \\ b \end{pmatrix}$ is parallel to $\begin{pmatrix} a \\ b \end{pmatrix}$

• Modulus of a vector:

- In general, if $x = \begin{pmatrix} m \\ n \end{pmatrix}$, $|x| = \sqrt{m^2 + n^2}$

7.2 Matrices

• Addition:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} a+p & b+q \\ c+r & d+s \end{pmatrix}$$

• Multiplication by scalar

$$k \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$$

• Multiplication by vector:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} ap+br & aq+bs \\ cp+dr & cq+ds \end{pmatrix}$$

- You can only multiply if no. of columns in left equals to no. of rows in right

• Determinant:

- Determinant = leading diagonal – secondary diagonal

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad |A| = (ad) - (bc)$$

• Inverse:

- To work out inverse, switch leading diagonal, negate secondary diagonal, multiply by $\frac{1}{|a|}$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad A^{-1} = \frac{1}{(ad-bc)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

7.3 Transformation

• Reflection (M):

- When describing a reflection, the position of the mirror line is essential.

• Rotation (R):

- To describe a rotation, the centre of rotation, the angle of rotation and direction of rotation are required.
- A clockwise rotation is negative and an anticlockwise rotation is positive.

• Translation (T):

- When describing a translation it is necessary to give the translation vector

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

• **Enlargement (E):**

- To describe an enlargement, state the scale factor, K and the centre of enlargement

$$\text{Scale factor} = \frac{\text{length of image}}{\text{length of object}}$$

$$\text{Area of image} = K^2 \text{ area of object}$$

- If $K > 0$, both object and image lie on same side of the centre of enlargement.
- If $K < 0$, object and image lie on opposite side of the centre of enlargement.

7.4 Transformation by Matrices

• **Reflection:**

- $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ Reflection in the x – axis
- $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ Reflection in the y – axis
- $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Reflection in the line $y = x$
- $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ Reflection in the line $y = -x$

• **Enlargement:**

- $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ where k =scale factor and centre of enlargement = (0,0)

• **Rotation:**

- $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ Rotation 90° anticlockwise, centre (0,0)
- $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ Rotation 90° clockwise, centre (0,0)
- $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ Rotation 180° clockwise/ anticlockwise, centre (0,0)

8. PROBABILITY

- Probability is the study of chance, or the likelihood of an event happening.

$$\text{Probability of an event} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

- If probability = 0, the event is impossible and if probability =1, the event is certain to happen
- All probabilities lie between 0 and 1.

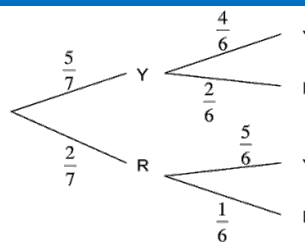
8.1 Events

Exclusive events:

- Two events are exclusive if they cannot occur at the same time.

• **The OR Rule:**

- For exclusive events A and B
- $p(A \text{ or } B) = p(A) + p(B)$



Independent events:

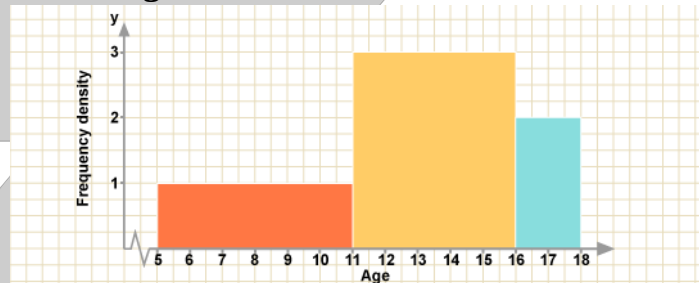
- Two events are independent if occurrence of one is unaffected by occurrence of other.

• **The AND Rule:**

- $p(A \text{ and } B) = p(A) \times p(B)$

9. STATISTICS

9.1 Histograms



- A histogram displays the frequency of either continuous or grouped discrete data in the form of bars.
- The bars are joined together.
- The bars can be of varying width.
- The frequency of the data is represented by the area of the bar and not the height.
- When class intervals are different it is the area of the bar which represents the frequency not the height
- Instead of frequency being plotted on the vertical axis, frequency density is plotted.

• **Class width** = Interval

• **Frequency density** = Height

$$\text{Frequency} = \text{Class width} \times \text{Frequency density}$$

9.2 Averages

• **Mean:**

$$\frac{\text{Sum of values}}{\text{number of values}}$$

• **Median:**

- The middle value when the data has been written in ascending or descending order

- Odd no. of values $\frac{5+1}{2} = 3rd \text{ value}$
- Even no. of values $\frac{6+1}{2} = 3.5th \text{ value}$
(add two values divide by 2)

• **Mode:**

- Most frequently occurring value

• **Range:**

- Difference between highest and lowest values

• **Estimated mean of grouped data:**

- Work out midpoints of each group and multiply by frequency
- Divide by number of values

9.3 Cumulative Frequency

- Cumulative frequency is the total frequency up to a given point.
- Inter-quartile range = upper quartile – lower quartile

